Timing Channels with Identical Quanta

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NSF BioCom² Workshop
Boston, November 8-9, 2012
A Biological Playground
1995 Nobel Prize in Medicine: Morphogenesis

Christiane Nusslein-Volhard

- Different signal concentrations $\rightarrow$ different gene expression

- Egg imposes initial signal gradients

- Extracellular diffusion $+$ neighbor interactions

- Developing cells receive/emit their own signals

- Embryogenesis and developmental magic ensues
A Confused Communication Theorist

- Gradient? mostly indirect evidence (gene splicing/silencing studies)
- Simple diffusion can be disrupted
- Abrupt anatomy change \(\neq\) abrupt gradient change
- Processing mechanisms abound (cells are COMPLEX machines!)
- Potential modeling morass
- **However:** cell fate is both discrete and amazingly reproducible

Organizing principle: digital info transfer/processing limits?
The Heroic Reductionist Picture

Signal manufacture takes energy

What can a cell tell the world?

What can groups of cells tell each other?

Energy + IT bounds \[\Rightarrow \text{(modeling morass)}\]
A Reductionist MiniMovie

http://www.winlab.rutgers.edu/~crose/DiffusionChannel.swf
Quanta Channels: timing

First Passage: \( D \) i.i.d. \( E[D_m] = 1/\lambda \ \forall m \)
Launch Constraints: \( T \in [0, T]^M \) or \( E[T_m] = \tau \)

We seek \( \max_{f_T(\cdot)} I(T; \tilde{S}) = \max h(\tilde{S}) - h(\tilde{S}|T) \)
Chris Fears Order Distributions I

- Assume hyper-symmetric $f_T()$

- Disallow singular first passage time densities

- Define permutation $\Omega$

$$\mathbf{S} \xrightarrow{\Omega} \mathbf{\tilde{S}}$$

- Note that:

$$\{\mathbf{\tilde{S}}, \Omega\} \leftrightarrow \mathbf{S}$$
Since “folds” are zero measure:

\[ h(\tilde{S}) = h(S) - \log M! \]

\{\tilde{S}, \Omega\} ⇔ S implies

\[ h(S|T) = h(\Omega, \tilde{S}|T) = h(\tilde{S}|T) + H(\Omega|\tilde{S}, T) \]

or

\[ h(\tilde{S}|T) = h(S|T) - H(\Omega|\tilde{S}, T) \]

so that

\[ I(\tilde{S}; T) = h(S) + H(\Omega|\tilde{S}, T) - h(S|T) - \log M! \]

**Bye Bye (full blown) Order Distributions!**
“Tension” between $h(S)$ and $H(\Omega|\tilde{S}, T)$

$$\max_{f_T} I(\tilde{S}; T) = \max_{f_T} \left[ h(S) + H(\Omega|\tilde{S}, T) \right] - h(S|T) - \log M!$$

- Max $H(\Omega|\tilde{S}, T)$: always launch all quanta at same time
- Max $h(S)$: i.i.d. quanta launch
- Variational calculus constrained max of $h(S) + H(\Omega|\tilde{S}, T)$
  - utter failure (at least in Chris’ hands)
Bounding $I(\tilde{S}; T)$: $C_q$ and $C_t$

Information Theory Is Asymptotic

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<td>$\gamma(M)$</td>
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| \(\tau(M)\) | | \(2\tau(M)\) | \(k\tau(M)\)

- General Upper Bound: 
  - Obvious (data processing – nothing tighter yet)

- Capacity-per-unit-cost lower bounds
  - Fix $\rho = M/\tau$, define $\chi = \lambda/\rho$ and (carefully) take $M \to \infty$
  - Iff mean first passage time exists: $C_q$ and $C_t = \rho C_q$

- We have general and special case results (exponential first passage)
Lower Bounds
What Have We (not) Done?

- General approach to communication with identical quanta
- Subsumes concentration-based approaches
- General and special case bounds
- Have not applied to biological systems
Not Feeling Very Heroic Yet
Ever Optimistic “What’s Next?” Blueprint

- **Embryogenesis Modeling:** Networks of identical units with a “leader”
  - Networks of identical units who can repeat, collaborate & initiate

- **Key Questions:**
  - Who can know what when?
  - Are morphogenic gradients (concentrations) enough?
  - Does timing add enough?

- **Potential Tool:** *Network Equivalence* – Koetter/Effros/Medard

- **Shhhhh! Caveat:** It all depends critically on the simple problem bounds
Still Awake?

Thanks!